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Pi-Mode Structures -- Results and Implications for Operation*

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Abstract

Pi-mode structures are the type chosen for high-energy synchrotrons and for medium- and high-beta accelerating superconducting structures. A coupled-circuit analysis, allowing for errors in cell frequencies and cell coupling constants, has been used to determine relative on-axis fields between cells, operating frequencies, end-cell tuning, relative on-axis field phase shifts and field tilts when operated off resonance. Formulae are given for the above information as well as specific examples to show sensitivities and machining/assembly/control tolerances.

1 RLC Loop Coupled Circuit

Many rf cavities employed in high-energy synchrotrons and superconducting accelerators use pi-mode structures. Relative on-axis average electric fields in a multi-cell cavity can be investigated using coupled-circuit analysis. This analysis method was used effectively for many years, beginning with Dunn et al [1-4] and later at Los Alamos [5-6] and Chalk River [7]. LOOPER [8], a program written by the author to analyze coupled circuits for resonance characteristics, is available from the Los Alamos Code Group [9]. LOOPER was validated from multi-cell SUPERFISH [10] calculations; agreement to three significant figures for relative fields and quality factors, and nine for resonant frequencies. LOOPER also provides information on phase shifts along a structure, match to the drive(s) and beam coupling effects.

A coupled RLC circuit analog of a coupled-cell cavity with first-neighbor coupling between loops has loop frequency $\omega_0 = \sqrt{1/2LC}$, quality factor $Q = 2\omega_0 L/R$, cell-to-cell coupling k (the mutual inductance), and loop current amplitude i_n for cell average axial electric field.

The dispersion relationship for the coupled system is $f = f_O/\sqrt{1+k\cos\phi}$. Solutions for ϕ of a finite chain with N cells, and loop current amplitudes are:

End Cells	ϕ, Phase Value	i_n, Current Solution	q, Mode Numbering
Half	$\pi\dfrac{(q-1)}{(N-1)}$	$\approx\cos\left(\dfrac{(n-1)\pi(q-1)}{(N-1)}\right)$	$1,2,3\ldots,N$
Full	$\pi\dfrac{q}{(N+1)}$	$\approx\sin\left(\dfrac{n\pi q}{(N+1)}\right)$	$1,2,3\ldots,N$

Relative to the dispersion curve, modes with *0* and π phase values exist for half-cell terminated systems, but not for full-cell termination. However, the last mode, $\phi=\pi N/(N+1)$, is the only mode with π phase shifts between each cell; hence π mode. Similarly, for the first mode with only *0* phase shift between cells. All other modes have combinations of *0*, $\pi/2$ and π phase shifts between cells. Unlike for a half-cell terminated cavity, relative fields are not flat for the two end modes, *q = 1 or N*, for full-cell termination.

Relative field solutions are obtained from solving a set of N coupled equations using finite difference techniques where the nth loop voltage using Kirchoff's Law is given by $\left[R + j\left(2\omega L - \dfrac{1}{\omega C}\right)\right]i_n + j\omega kL\left(i_{n+1} + i_{n-1}\right) = 0$. End equations use *A=1/2* or *1* for half- or full-cell termination:

$$\left.\begin{aligned}AKi_1 + i_2 &= 0\\ AKi_N + i_{N-1} &= 0\end{aligned}\right) K = \frac{2}{k}\left(1 - \left(\frac{\omega_0}{\omega}\right)^2 - j\frac{\omega_0}{\omega Q}\right) \tag{1}$$

For flat fields in the *0* or π mode with full-cell termination, the end equations must be modified (to be similar to those with half-cell termination) by having end cell $f_{end} = f_0\sqrt{(1-\sigma k/2)/(1-\sigma k)}$ with $\sigma = 1$ for the π mode, and $\sigma = -1$ for the *0* mode. When one of these end modes is made flat, the relative field distributions change for the rest of the modes. The π mode frequency has correction factors that account for Q and N giving $f_\pi = \dfrac{f_0}{\sqrt{1-k(1+\beta)}}$ where $\beta = \dfrac{5(N-1)^2}{8k^2Q^2}$, the π mode frequency correction factor. This factor was confirmed from LOOPER calculations with error-free systems to be accurate within nine significant figures for k's from 1% to 6%, cell numbers to nine and Q's as low as 10,000.

2 Step-Wise Solutions of the π Mode Equations

Although solutions are possible from analogs such as LOOPER, or from SUPERFISH and/or MAFIA [11], sometimes it is worthwhile to understand the performance of a coupled cavity using analytic formulae. In addition, formulae give insight into system performance and consequences of particular parameter changes. For these reasons, the equations were solved stepwise from the first to the last with a drive e_N in cell N. The last equation becomes $AKi_N + i_{N-1} = e_N/jk\omega L$. Solutions below were found to agree with LOOPER calculations to within three significant figures. Differences in cell parameters (from assembly perturbations, machining tolerances, etc.) are represented by different cell values: the nth cell frequency, $f_{0n} = f_0(1+\Delta_n)$ and coupling between the nth and the n+1th cell, $k_{n,n+1} = k(1+\varepsilon_{n,n+1})$. Neglecting higher order terms gives the following for the nth cell in an N cell chain in terms of defining the first cell, $i_1 = 1 + j0$:

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$$i_n(full)/(-1)^{n-1} \cong 1+n(n-1)\beta - \frac{(n+1)n(n-1)(n-2)}{6k^2Q^2}\left[1-k(1+\beta)+\frac{2(n-3)(n+2)\beta}{15}+\frac{k}{(n+1)}\right]\cdots$$
$$-\frac{4}{k}\left[(n-1)\Delta_1+\cdots+3\Delta_{n-3}+2\Delta_{n-2}+\Delta_{n-1}\right]-(\cdots+7\varepsilon_{n-4,n-3}+5\varepsilon_{n-3,n-2}+3\varepsilon_{n-2,n-1}+\varepsilon_{n-1,n})\cdots \quad (2)$$
$$-j\frac{n(n-1)}{kQ}\left[1-\frac{k(1+\beta)}{2}+\frac{(n-2)(n+1)\beta}{3}+\frac{k}{2n}\right] \quad \text{where } \beta=\frac{5(N-1)^2}{8k^2Q^2}\text{, the } \pi \text{ mode frequency correction factor.}$$

$$i_n(half)/(-1)^{n-1} \cong 1+\beta(n-1)^2-\frac{n(n-1)^2(n-2)}{6k^2Q^2}\left[1-k(1+\beta)+\frac{2(n-3)(n+1)\beta}{15}\right]\cdots$$
$$-\frac{4}{k}\left[\frac{(n-1)}{2}\Delta_1+\cdots+3\Delta_{n-3}+2\Delta_{n-2}+\Delta_{n-1}\right]-(\cdots+7\varepsilon_{n-4,n-3}+5\varepsilon_{n-3,n-2}+3\varepsilon_{n-2,n-1}+\varepsilon_{n-1,n})\cdots \quad (3)$$
$$-j\frac{(n-1)^2}{kQ}\left[1-\frac{k}{2}(1+\beta)+\frac{n(n-2)\beta}{3}\right]$$

Field tilt depends on the square of cell number, and is linear to frequency and coupling constant errors.

For most applications Eq. 2 can be simplified to $i_n(full)/(-1)^{n-1} \cong 1+n(n-1)\beta - jn(n-1)/kQ$, showing that the phase shift difference in the average on-axis fields from end cell to drive cell is given by $\Delta\phi_N(full) \cong -N(N-1)/kQ$ in radians for small values, useful information not easily obtained from SUPERFISH.

3 Results of Calculations for the π Mode

Extensive information for designing, constructing and operating pi-mode structures is available, especially for superconducting cavities [12-14]. This report provides additional information to assist sensitivity understanding.

3.1 Off-Resonance Field Tilt

Differentiation of Eq. 2 with respect to β yields: $\Delta i_n(full)/(-1)^{n-1} \cong 2n(n-1)(1-k)\Delta f_\pi / kf_\pi$, an equation independent of Q that has been validated with LOOPER simulations. In addition to the usual considerations for on-resonance control and for Lorentz force effects, a five-cell, 1% k, cavity with required field tolerance of $\pm$ ½%, a frequency tolerance $\Delta f / f$ is $\pm 1.25 \times 10^{-6}$ or $\pm$ 1 kHz at 800 MHz is needed. The tilt in field changes direction, as expected, on either side of resonance, introducing an interesting aspect to some control algorithms.

3.2 Frequency and Coupling Errors in the Cells

Eqs. 2 and 3 show that an error in cell-to-cell coupling, $\varepsilon_{n-1,n}$, or in frequency, Δ_n, propagates throughout the entire cavity fields and is independent of Q. To minimize field errors, a frequency error in the n^{th} cell requires adjacent cells to have opposite sign errors, one-half the n^{th} value. This correction yields field distributions almost as flat as if there were no errors in the cavity, except in the local cell, n. A complex relationship involving all constants must be satisfied to achieve fields almost as flat as those without coupling errors.

Propagating through the entire system means, for example, a Δ / k or ε error of 1% yields field errors in a typical five-cell cavity (k=0.01, loaded Q=60,000) along the length of from 2% up to 8% and from 1% up to 3%, respectively, depending on the location of the error.

Resonance shifts df_π(Hz) = f_π(Hz)$k\epsilon_{n,n-1}$/[2(1-k)N] and df_π(Hz) = $(f_\pi/f_0)\Delta_n f_0$(Hz)/N for coupling constant and frequency errors, respectively.

3.3 End Cell Tuning and Coupling Constants

A series of SUPERFISH calculations were completed to determine parameters for end cell tuning as a function of cell beta and beam-bore hole, the latter of which affects the coupling constant. One easy method to obtain flat fields is to make the end beam hole larger than the beam bore hole for at least one end beam hole diameter in length away from the end cell. Best fit to the data was [end-radius(cm-GHz) = 1.24*bore-radius(cm-GHz)-0.46]. For instance, a 3 cm bore has a 3.26 cm end bore at 1 GHz, while a 1 cm bore has a 1.08 cm end bore at 3 GHz.

As described, there are many advantages for a coupling constant, k, as high as possible. In the design process for a superconducting cavity, many variables are considered as described in reference 12. "k" was determined as a function of aperture bore radius for different cell betas.

In order to maintain relative cell fields to within ½%, for a five-cell, 1% k, cavity, coupling constant errors need to be within 1/6% cell to cell. Tolerances on aperture dimensions vary from about 0.0013 cm for a 0.5 beta cell to 0.0016 cm for a 1.0 beta cell, attainable tolerances.

3.3 Dispersion Curve Characteristics

Detuning end cells of a full-cell terminated cavity has a significant effect on the dispersion curve. An inspection of dispersion curves shows extreme sensitivity between the π mode and the next nearest mode. The change between these two modes is most noticeable as the frequency of the end cell changes. The smaller the number of cells, the more exaggerated the curve becomes.

This changing dispersion curve pattern can be used to estimate or to infer π mode field distributions in a cavity by observing mode frequencies as a function of actions. Straight-line fits to LOOPER data give coupling constants as a function of [f(N)-f(N-1)], the frequency difference between the two last modes. For a 5-cell cavity, $k = (0.5/f_0)\{0.019[f(N) - f(N-1)] + 0.0013\}$, where f_0 is the cell frequency in GHz. For scaled KfA data with [f(N)-f(N-1)] difference of 2 MHz at 1 GHz, k is 1.96%.

Similarly, straight-line fits give coupling constant as a function of [f(N-1)-f(1)], frequency difference between the second last mode and the first mode. For the above case, $k = (0.5/f_0)\{0.0025[f(N-1) - f(1)] + 0.00007\}$. For a scaled [f(N-1)-f(1)] difference of 15 MHz at 1 GHz, k is 1.94%. The ratio of these two values can be used to estimate field flatness in the cavity. In this case, fields are flat to within 1% (1.96/1.94).

3.4 Using Dispersion Curve Characteristics for Field Pattern Changes

A difficulty in building and tuning a superconducting cavity is that the on-axis field pattern is well known at room temperature, but is only assumed to remain the same at cold temperatures. The following method provides an indirect means for inferring changed fields; based on the room temperature measurements, the mode spectra at cold temperature and a simple assumption. It is assumed that end cells (because of connections to end bore tubes and mechanical systems) behave in a manner different from that of the middle cells. Field change along the cavity length can be estimated by measuring cavity modes.

From mode frequency measurements, an important dimensionless dispersion curve ratio is determined, {[f(N-1)-f(1)]/[f(N)-f(N-1)]}. A change in this ratio is used to infer a change in the field flatness ratio. LOOPER calculations have shown that the mode spectra ratio is identical to the field ratio for changes <10%.

4 Summary

A number of interesting trends have been determined for pi-mode coupled cavities using coupled-circuit theory and solutions to various parametric variations. Analysis has shown advantages for coupling constants as high as possible ($k \geq 2\%$) within constraints of other parameters and conditions. Cavity cell number should be small ($N \leq 5$) to minimize error-related effects. A method for inferring field flatness when a cavity is operated different from that when fields were measured has been developed. Experimental verification is needed before this method is fully accepted. Another report, being published by the author, provides more details and useful graphs.

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